GCE AS MARKING SCHEME

SUMMER 2022

AS (NEW)
FURTHER MATHEMATICS
UNIT 1 FURTHER PURE MATHEMATICS A
2305U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 1 FURTHER PURE MATHEMATICS A

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1. a) i)	METHOD 1: $zw = (3-4\mathrm{i})(2-\mathrm{i}) = 6 - 3\mathrm{i} - 8\mathrm{i} + 4\mathrm{i}^2$ $zw = 2 - 11\mathrm{i}$	B2	B1 for unsimplified expansion with 3 correct terms
	$\lvert zw\rvert = \sqrt{2^2 + (-11)^2} = 5\sqrt{5}$	B1	FT their zw (zw must be seen)
	$\arg zw = \tan^{-1}\left(-\frac{11}{2}\right) = -1.39$ or -79.7°	B1	oe FT their zw if not in 1st quadrant
	METHOD 2: $\lvert z\rvert = \sqrt{3^2 + (-4)^2} = 5$ $\lvert w\rvert = \sqrt{2^2 + (-1)^2} = \sqrt{5}$	(B1)	Both mods
	$\arg z = \tan^{-1}\left(-\frac{4}{3}\right) = -0.927$ or -53.13° $\arg w = \tan^{-1}\left(-\frac{1}{2}\right) = -0.464$ or -26.57°	(B1)	oe Both args oe
	Therefore, $\lvert zw\rvert = 5 \times \sqrt{5} = 5\sqrt{5}$	(B1)	FT args and mods
	$\arg zw = -0.927 + -0.464 = -1.39$ or -79.7°	(B1)	oe FT args and mods (mods and args must be seen)
		[4]	
ii)	$\therefore 5\sqrt{5}(\cos(-1.39) + \mathrm{i}\sin(-1.39))$ OR $5\sqrt{5}(\cos(-79.7^o) + \mathrm{i}\sin(-79.7^o))$	B1 [1]	oe FT their mod and arg
b)	METHOD 1: $\frac{1}{v} = \frac{1}{2-\mathrm{i}} - \frac{1}{3-4\mathrm{i}}$		
	$\frac{1}{v} = \frac{3-4\mathrm{i}-2+\mathrm{i}}{(3-4\mathrm{i})(2-\mathrm{i})}$	M1	Attempt to combine
	$\frac{1}{v} = \frac{1-3\mathrm{i}}{2-11\mathrm{i}}$	A1	
	$v = \frac{2-11\mathrm{i}}{1-3\mathrm{i}}$	A1	
	$v = \frac{2-11\mathrm{i}}{1-3\mathrm{i}} \times \frac{1+3\mathrm{i}}{1+3\mathrm{i}}$	M1	FT their v M0 for no working
	$v = \frac{35-5\mathrm{i}}{10}\left(= \frac{7-\mathrm{i}}{2}\right)$		
	$v = 3.5 - 0.5\mathrm{i}$	A1	oe cao
	METHOD 2: $\frac{1}{v} = \frac{1}{2-\mathrm{i}} - \frac{1}{3-4\mathrm{i}}$		

	$\frac{1}{v} = \frac{z-w}{zw}$	(M1)	Attempt to combine
	$v = \frac{zw}{z-w}$ or $\frac{1}{v} = \frac{1-3\mathrm{i}}{2-11\mathrm{i}}$	(A1)	
	$v = \frac{2-11\mathrm{i}}{1-3\mathrm{i}}$	(A1)	
	$v = \frac{2-11\mathrm{i}}{1-3\mathrm{i}} \times \frac{1+3\mathrm{i}}{1+3\mathrm{i}}$	(M1)	FT their v M0 no working
	$v = \frac{35-5\mathrm{i}}{10}\left(= \frac{7-\mathrm{i}}{2}\right)$ $v = 3.5 - 0.5\mathrm{i}$	(A1)	oe cao
	METHOD 3: Attempt to realise at least one fraction e.g. $\frac{1}{2-\mathrm{i}} \times \frac{2+\mathrm{i}}{2+\mathrm{i}}$ OR $\frac{1}{3-4\mathrm{i}} \times \frac{3+4\mathrm{i}}{3+4\mathrm{i}}$	(M1)	M0 no working
	$\frac{1}{v} = \frac{2+\mathrm{i}}{5} - \frac{3+4\mathrm{i}}{25}$		
	$\frac{1}{v} = \frac{7+\mathrm{i}}{25}$	(A1)	
	$v = \frac{25}{7+\mathrm{i}}$	(A1)	
	$v = \frac{25}{7+\mathrm{i}} \times \frac{7-\mathrm{i}}{7-\mathrm{i}}$	(M1)	FT their v M0 no working
	$v = \frac{35-5\mathrm{i}}{10}\left(= \frac{7-\mathrm{i}}{2}\right)$ $v = 3.5 - 0.5\mathrm{i}$	(A1) [5]	oe cao
c)	$\bar{v} = \frac{7+\mathrm{i}}{2}$	B1	FT their v provided complex
	$v\bar{v} = \frac{7-\mathrm{i}}{2} \times \frac{7+\mathrm{i}}{2} = \frac{25}{2}$	B1 [2]	oe
		[12]	

2.a)	METHOD 1: Let $X = \begin{pmatrix} a \\ b \end{pmatrix}$ $\begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -11 \\ 7 \end{pmatrix}$ Therefore, $3a + 4b = -11$ $-a - 2b = 7$ Solving, $a = 3$ and $b = -5$ $X = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ METHOD 2: $det\ A = (3 \times -2) - (4 \times -1) = -2$ $A^{-1} = \frac{1}{-2}\begin{pmatrix} -2 & -4 \\ 1 & 3 \end{pmatrix}$ Therefore, $X = A^{-1}B = \frac{1}{-2}\begin{pmatrix} -2 & -4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -11 \\ 7 \end{pmatrix}$ $X = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$	M1 A1 M1 A1 (B1) (B1) (M1) (A1) [4]	Attempt to form 2 sim eqns Attempt to solve Must be in matrix form si Must be in matrix form
b) (i)	If reflection in $y = -2x$, then $\tan\theta = -2$ $\therefore\ \theta = \tan^{-1}(-2)$ Reflection matrix: $\begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$	B1 B2 [3]	si B1 for 1 error (possibly repeated) If B2 then -1 for PA
b) (ii)	METHOD 1: Therefore, $EF = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 7 & 1 \end{pmatrix}$ $EF = \begin{pmatrix} -\frac{34}{5} & -\frac{13}{5} \\ \frac{13}{5} & -\frac{9}{5} \end{pmatrix}$ Midpoint: $\left(-\frac{47}{10}, \frac{2}{5}\right)$ METHOD 2: Midpoint of *CD* = $\left(\frac{2+3}{2}, \frac{7+1}{2}\right) = \left(\frac{5}{2}, 4\right)$ Therefore,	 M1 A1 A1 B1 (B1)	FT their *T* For attempt to multiply at least 1 point matrix Left column Right column May be seen as separate matrices oe, FT their *E* and *F* FT their *T*

	$\begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}\begin{pmatrix} \frac{5}{2} \\ 4 \end{pmatrix}$ $= \begin{pmatrix} -\frac{47}{10} \\ \frac{2}{5} \end{pmatrix}$ Midpoint of *EF*: $\left(-\frac{47}{10}, \frac{2}{5}\right)$	(M1) (A1) (A1) [4]	FT their midpoint oe
		[11]	
3.	$x = -1 + 4\lambda \quad y = 2 - 2\lambda \quad z = -6 + 7\lambda$ Substituting, $\therefore\ -3 + 12\lambda + 16 - 16\lambda + 54 - 63\lambda = 0$ $67 - 67\lambda = 0$ $\lambda = 1$ $\therefore x = 3 \quad y = 0 \quad z = 1$ $\Rightarrow\ (3, 0, 1)$	B1 M1 A1 A1 B1 **[5]**	si FT their λ and their x, y, z provided at least 2 correct
4.	$1^2 + 2^2 + 3^2 + \cdots + N^2$ can be written as $\sum_{r=1}^{N} r^2$ $\sum_{r=1}^{N} r^2 = (3N-2)^2$ $\frac{1}{6}N(N+1)(2N+1) = 9N^2 - 12N + 4$ $2N^3 + 3N^2 + N = 54N^2 - 72N + 24$ $2N^3 - 51N^2 + 73N - 24 = 0$ Finding one factor, eg. $(N-1)$ $\therefore\ (N-1)(2N^2 - 49N + 24) = 0$ $\therefore (N-1)(2N-1)(N-24) = 0$ $\therefore\ N = 1 \text{ or } N = \frac{1}{2} \text{ or } N = 24$ Therefore, $N = 1, 24$	 M1 A1 A1 B1 m1 A1 A1 **[7]**	 cao $(N-k)$ form Linear × Quadratic (2 terms correct) Must reject $N = \frac{1}{2}$

5. a)	$\lvert z-3+2\text{i}\rvert = \lvert z-3\rvert$ $\lvert x+\text{i}y-3+2\text{i}\rvert = \lvert x+\text{i}y-3\rvert$ $\lvert(x-3)+\text{i}(y+2)\rvert = \lvert(x-3)+\text{i}y\rvert$ $\sqrt{(x-3)^2+(y+2)^2} = \sqrt{(x-3)^2+y^2}$ $x^2-6x+9+y^2+4y+4 = x^2-6x+9+y^2$ $4y+4=0$ $y=-1$	M1 m1 A1 [3]	 oe Mark final answer Sight of answer only M1m1A1
b)	It is the perpendicular bisector of the line joining the points **(3, -2)** and **(3, 0)** OR The locus of P is all the points which are equidistant from **(3, -2)** and **(3,0)**.	B1 (B1) [1]	
		[4]	
6.	$\alpha+\beta+\gamma = -\frac{p}{2}$ $\alpha\beta+\beta\gamma+\gamma\alpha = -63$ $\alpha\beta\gamma = -\frac{q}{2}$ Let initial root be α AND use of g.p. property Then other roots are -3α and 9α Therefore $\left(7\alpha = -\frac{p}{2}\right)$ $-21\alpha^2 = -63$ $\left(-27\alpha^3 = -\frac{q}{2}\right)$ $\therefore\ \alpha^2 = 3 \quad \Rightarrow \quad \alpha = \pm\sqrt{3}$ If $\alpha = +\sqrt{3}$, $p = -14\sqrt{3}$ and $q = 162\sqrt{3}$ AND If $\alpha = -\sqrt{3}$, $p = 14\sqrt{3}$ and $q = -162\sqrt{3}$	B1 B1 B1 M1 A1 A1 A1 A1 **[8]**	May be seen later in working Accept solutions where α,β,γ interchanged oe (e.g. $-3\alpha, 9\alpha, -27\alpha$) provided M1 awarded cao

7. a)	From lines L_1, L_2: $(2 \times 3) + (1 \times n) + (1 \times -3) = 0$ $6 + n - 3 = 0$ $n = -3$ From lines L_1, L_3: $(2 \times p) + (-3 \times 3) + (1 \times 4) = 0$ $p = \frac{5}{2}$	M1 A1 (M1) A1 [3]	 convincing If not awarded for L_1, L_2
b)	$\left(3 \times \frac{5}{2}\right) + (1 \times 3) + (-3 \times 4) = -\frac{3}{2}$ $\lvert 3\mathbf{i} + \mathbf{j} - 3\mathbf{k} \rvert = \sqrt{19}$ $\left\lvert \frac{5}{2}\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} \right\rvert = \sqrt{\frac{125}{4}}$ Therefore, $\cos\theta = \frac{-\frac{3}{2}}{\sqrt{19}\sqrt{\frac{125}{4}}}$ $\theta = 93.5^o$ Therefore, acute angle is $\theta = 86.5^o$	B1 B1 M1 A1 [4]	si FT their p for B1B1M1 si Both mods oe cao
		[7]	

8.	Rotation matrix: $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	B1	
	$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1	Attempt to multiply Allow 1 error (possibly repeated)
	$= \begin{pmatrix} \frac{1}{2}x - \frac{\sqrt{3}}{2}y \\ \frac{\sqrt{3}}{2}x + \frac{1}{2}y \\ z \end{pmatrix}$	A1	
	Therefore, $x' = \frac{1}{2}x - \frac{\sqrt{3}}{2}y$ $y' = \frac{\sqrt{3}}{2}x + \frac{1}{2}y$		
	$\therefore\ \frac{1}{2}x - \frac{\sqrt{3}}{2}y = \frac{\sqrt{3}}{2}x + \frac{1}{2}y$ $x - \sqrt{3}y = \sqrt{3}x + y$ $x - \sqrt{3}x = y + \sqrt{3}y$ $x(1-\sqrt{3}) = y(1+\sqrt{3})$	M1	FT their images matrix
	$y = \frac{x(1-\sqrt{3})}{1+\sqrt{3}}$	A1	cao
	$\frac{1-\sqrt{3}}{1+\sqrt{3}} = \frac{(1-\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$ $= \frac{1+3-\sqrt{3}-\sqrt{3}}{1-3+\sqrt{3}-\sqrt{3}} = \frac{4-2\sqrt{3}}{-2}$	M1	M0 no working FT their y of equivalent difficulty e.g. $y = \frac{x(a+\sqrt{b})}{c+\sqrt{d}}$
	$y = (-2+\sqrt{3})x$	A1 **[7]**	

9. a)	$\frac{1}{r+1}-\frac{2}{r+2}+\frac{1}{r+3}$ $\frac{(r+2)(r+3)-2(r+1)(r+3)+(r+1)(r+2)}{(r+1)(r+2)(r+3)}$ $\frac{r^2+5r+6-2r^2-8r-6+r^2+3r+2}{(r+1)(r+2)(r+3)}$ $=\frac{2}{(r+1)(r+2)(r+3)}.$	M1 A1 [2]	 Convincing
b)	$\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{2}{5}+\frac{1}{6}\right)+\cdots$ $+\left(\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right)+\left(\frac{1}{n+1}-\frac{2}{n+2}+\frac{1}{n+3}\right)$ $=\frac{1}{2}-\frac{2}{3}+\frac{1}{3}$ $+\frac{1}{n+2}-\frac{2}{n+2}+\frac{1}{n+3}$ $=\frac{1}{6}-\frac{n+3-n-2}{(n+2)(n+3)}$ $=\frac{1}{6}-\frac{1}{(n+2)(n+3)}$	M1 A1 A1 A1 A1 [5]	Substituting values – At least three correct sets of brackets Must have at least one correct algebraic set of brackets Convincing
c)	$\sum_{r=1}^{5} A_r=\frac{1}{6}-\frac{1}{7\times 8}=\frac{25}{168}$ AND $\sum_{r=1}^{10} A_r=\frac{1}{6}-\frac{1}{12\times 13}=\frac{25}{156}$ $\frac{25}{168}:\frac{25}{156}$ $13:14$	B1 B1 [2]	Both
		[9]	